Using PETSc Solvers in PyLith

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We want to enable users to assess solver performance, and optimize solvers for particular problems.
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What do we care about?

- Error
  The difference between my computed answer and the true solution of my equations

- Residual
  How close my computed answer comes to satisfying my equations
What do we care about?

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What do we care about?

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  The difference between my computed answer and the true solution of my equations

- **Residual**
  How close my computed answer comes to satisfying my equations
This gives us two metrics for measuring the quality of our computed solutions.

They are not the same, but they are related.
This gives us two metrics for measuring the quality of our computed solutions.

\[
\frac{\|u - u^*\|}{\|u^*\|} \leq \kappa \frac{\|F(u)\|}{\|F(0)\|}
\]
This gives us two metrics for measuring the quality of our computed solutions.

For linear equations, $F(u) = Au - b$, this becomes

$$\frac{\|u - u^*\|}{\|u^*\|} = \frac{\|A^{-1}(Au - b)\|}{\|A\|\|A^{-1}\|\|Au - b\|} \leq \frac{\|A\|\|A^{-1}\|\|Au - b\|}{\|b\|} \leq \kappa(A)\frac{\|Au - b\|}{\|b\|}$$
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For linear equations, $F(u) = Au - b$, this becomes

$$\kappa(A) = \|A\|\|A^{-1}\|$$
Outline

1. Mathematical Background
2. Controlling the Solver
3. Where do I begin?
4. How do I improve?
5. Can We Do It?
6. Nonlinear Systems
All of PETSc can be controlled by options

- ksp_type gmres
- start_in_debugger

All objects can have a prefix, velocity_pc_type jacobi

All PETSc options can be given to PyLith

--petsc.ksp_type=gmres
--petsc.start_in_debugger
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All PETSc options can be given to PyLith

--petsc.ksp_type=gmres
--petsc.start_in_debugger
Examples

We will illustrate options using

PETSc SNES ex19, located at
$PETSC_DIR/src/snes/examples/tutorials/ex19.c

and

PyLith Example hex8, located at
$PYLITH_DIR/examples/3d/hex8/
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Where do I begin?

What solvers can I choose from?

- **Direct (LU, Cholesky)**
  - Robust, large memory and time

- **Multigrid**
  - Touchy, small memory and time

- **Domain Decomposition**
  - Somewhat robust, medium memory and time

- **Krylov**
  - Ineffectual alone, small memory and time
What is a Krylov solver?

A Krylov solver builds a small model of a linear operator $A$, using a subspace defined by

$$\mathcal{K}(A, r) = \text{span}\{r, Ar, A^2r, A^3r, \ldots\}$$

where $r$ is the initial residual.

The small system is solved directly, and the solution is projected back to the original space.
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$$\mathcal{K}(A, r) = \text{span}\{r, Ar, A^2r, A^3r, \ldots\}$$

where $r$ is the initial residual.

The small system is solved directly, and the solution is projected back to the original space.
Strength is *adaptivity*
- They can handle *low-mode* errors

They are *not* stand-alone solvers
- They need preconditioners

Scalability suffers after many iterates
- They do a lot of inner products
A preconditioner $M$ changes a linear system,

$$M^{-1}Ax = M^{-1}b$$

so that the effective operator is $M^{-1}A$, which is hopefully **better** for Krylov methods.

- Preconditioner should be inexpensive
- Preconditioner should accelerate convergence
What is a Preconditioner?

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Always start with LU:

- No iterative tolerance
- (Almost) no condition number dependence
- Check for accidental singularity

In parallel, you need a 3rd party package

- MUMPS (--download-mumps)
- SuperLU (--download-superlu_dist)
Always start with LU

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- No iterative tolerance
- (Almost) no condition number dependence
- Check for accidental singularity

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What if LU fails?

LU will fail for

- Singular problems
- Saddle-point problems

For saddles use `PC_FIELDSPLIT`

- Separately solves each field
- Decomposition is automatic in PyLith
- Autodetect with `-pc_fieldsplit_detect_saddle_point`
- Exact with full Schur complement solve
Where do I begin?

What if LU fails?

LU will fail for
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- Separately solves each field
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How do I improve?

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   - Look at what you have
   - Back off in steps
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How do I improve?

- Look at what you have
- Back off in steps
Use `-snes_view` or `-ksp_view` to output a description of the solver:

KSP Object: (fieldsplit_0_) 1 MPI processes
  type: fgmres
    GMRES: restart=100, using Classical (unmodified) Gram-Schmidt Orthogonalization with no iterative refinement
    GMRES: happy breakdown tolerance 1e-30
  maximum iterations=1, initial guess is zero
  tolerances:  relative=1e-09, absolute=1e-50,
    divergence=10000
  right preconditioning
  has attached null space
  using UNPRECONDITIONED norm type for convergence test
What did the convergence look like?

Use `-snes_monitor and -ksp_monitor`, or `-log_summary`:
What did the convergence look like?

Use \texttt{-snes\_monitor} and \texttt{-ksp\_monitor}, or \texttt{-log\_summary}:

0  SNES Function norm 0.207564
1  SNES Function norm 0.0148968
2  SNES Function norm 0.000113968
3  SNES Function norm 6.9256e-09
4  SNES Function norm < 1.e-11
Use `-snes_monitor` and `-ksp_monitor`, or `-log_summary`:

0 KSP Residual norm 1.61409
  Residual norms for mg_levels_1_ solve.
  0 KSP Residual norm 0.213376
  1 KSP Residual norm 0.0192085
Residual norms for mg_levels_2_ solve.
0 KSP Residual norm 0.223226
1 KSP Residual norm 0.0219992
  Residual norms for mg_levels_1_ solve.
  0 KSP Residual norm 0.0248252
  1 KSP Residual norm 0.0153432
Residual norms for mg_levels_2_ solve.
0 KSP Residual norm 0.0124024
1 KSP Residual norm 0.0018736
1 KSP Residual norm 0.02282
Use `-snes_monitor` and `-ksp_monitor`, or `-log_summary`:

<table>
<thead>
<tr>
<th>Event</th>
<th>Count</th>
<th>Time (sec)</th>
<th>Flops</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max Ratio</td>
<td>Max</td>
<td>Ratio</td>
</tr>
<tr>
<td>KSPSetUp</td>
<td>12</td>
<td>3.6259e-03</td>
<td>1.0</td>
<td>0.00e+00</td>
</tr>
<tr>
<td>KSPSolve</td>
<td>3</td>
<td>4.8937e-01</td>
<td>1.0</td>
<td>8.93e+05</td>
</tr>
<tr>
<td>SNESSSolve</td>
<td>1</td>
<td>4.9477e-01</td>
<td>1.0</td>
<td>9.22e+05</td>
</tr>
</tbody>
</table>
Use `-log_summary`:

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (sec)</th>
<th>Flops</th>
<th>---</th>
<th>Global ---</th>
<th>Total</th>
</tr>
</thead>
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<td></td>
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<td>Max Ratio</td>
<td>%T</td>
<td>%f</td>
<td>%M</td>
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<td>2.49e+04</td>
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<td>0</td>
<td>3</td>
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<tr>
<td>MatMult</td>
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<td>2.65e+05</td>
<td>1.0</td>
<td>0</td>
<td>29</td>
</tr>
<tr>
<td>PCAppl</td>
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<td>7.78e+05</td>
<td>1.0</td>
<td>58</td>
<td>84</td>
</tr>
<tr>
<td>KSPSetUp</td>
<td>3.6259e-03</td>
<td>0.00e+00</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>KSPSolve</td>
<td>4.8937e-01</td>
<td>8.93e+05</td>
<td>1.0</td>
<td>61</td>
<td>97</td>
</tr>
<tr>
<td>SNES Solve</td>
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<td>9.22e+05</td>
<td>1.0</td>
<td>62</td>
<td>92</td>
</tr>
</tbody>
</table>

Use `-log_view ::ascii_info_detail` to get this information as a Python module.
Look at what you have

How do I improve?

Use `-log_summary:

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (sec)</th>
<th>Flops</th>
<th>---</th>
<th>Global ---</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Ratio</td>
<td>Max</td>
<td>Ratio</td>
<td>%T</td>
</tr>
<tr>
<td>VecMDot</td>
<td>1.8904e-03</td>
<td>1.0</td>
<td>2.49e+04</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>MatMult</td>
<td>2.1026e-03</td>
<td>1.0</td>
<td>2.65e+05</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>PCApply</td>
<td>4.6001e-01</td>
<td>1.0</td>
<td>7.78e+05</td>
<td>1.0</td>
<td>58</td>
</tr>
<tr>
<td>KSPSetUp</td>
<td>3.6259e-03</td>
<td>1.0</td>
<td>0.00e+00</td>
<td>0.0</td>
<td>0</td>
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<tr>
<td>KSPSolve</td>
<td>4.8937e-01</td>
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<td>1.0</td>
<td>61</td>
</tr>
<tr>
<td>SNES Solve</td>
<td>4.9477e-01</td>
<td>1.0</td>
<td>9.22e+05</td>
<td>1.0</td>
<td>62100</td>
</tr>
</tbody>
</table>

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4 How do I improve?
   - Look at what you have
   - Back off in steps
GMRES $\Rightarrow$ BiCGStab

- `-ksp_type bcgs`
- Less storage
- Fewer dot products (less work)
- Variants `-ksp_type bcgsl` and `-ksp_type ibcgs`

Complete Table of Solvers and Preconditioners
Weaken the PC

LU $\Rightarrow$ ILU

- `-pc_type ilu`

- Less storage and work

In parallel,

- Hypre `-pc_type hypre -pc_hypre_type euclid`

- Block-Jacobi `-pc_type bjacobi -sub_pc_type ilu`

- Additive Schwarz `-pc_type asm -sub_pc_type ilu`

Default for MG smoother is Chebychev/SOR(2)
Weaken the PC

LU $\Rightarrow$ ILU

- $\text{pc\_type ilu}$

Less storage and work

In parallel,

- Hypre $\text{pc\_type hypre \ -pc\_hypre\_type euclid}$
- Block-Jacobi $\text{pc\_type bjacobi \ -sub\_pc\_type ilu}$
- Additive Schwarz $\text{pc\_type asm \ -sub\_pc\_type ilu}$

Default for MG smoother is Chebychev/SOR(2)
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LU $\Rightarrow$ ILU
- $-pc_{-type}$ ilu
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In parallel,
- Hypre $-pc_{-type}$ hypre $-pc_{-hypre_{-type}}$ euclid
- Block-Jacobi $-pc_{-type}$ bjacobi $-sub_{-pc_{-type}}$ ilu
- Additive Schwarz $-pc_{-type}$ asm $-sub_{-pc_{-type}}$ ilu

Default for MG smoother is Chebychev/SOR(2)
Can solve elliptic problems
- Laplace, elasticity, Stokes

Works for unstructured meshes

- `pc_type gamg`, `-pc_type ml`,
  `-pc_type hypre` `-pc_hypre_type boomeramg`

**CRUCIAL** to have an accurate near-null space
- `MatSetNearNullSpace()`
- PyLith provides this automatically

Use `-pc_mg_log` to put timing in its own log stage
How do I improve?

Back off in steps

PC_FieldSplit

- Separate solves for block operators
  - Physical insight for subsystems
  - Have optimal PCs for simpler equations
  - Suboptions $fs_{\text{fieldsplit}}_0_*$

- Flexibly combine subsolves
  - Jacobi: $fs_{\text{pc_fieldsplit}_\text{type}} = \text{additive}$
  - Gauss-Siedel: $fs_{\text{pc_fieldsplit}_\text{type}} = \text{multiplicative}$
  - Schur complement: $fs_{\text{pc_fieldsplit}_\text{type}} = \text{schur}$
The common block preconditioners for Stokes require only options:

The Stokes System

$$\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix}$$
The common block preconditioners for Stokes require only options:

- `pc_type fieldsplit`
- `pc_field_split_type additive`
- `fieldsplit_0_pc_type ml`
- `fieldsplit_0_ksp_type preonly`
- `fieldsplit_1_pc_type jacobi`
- `fieldsplit_1_ksp_type preonly`

\[
\begin{pmatrix}
\hat{A} & 0 \\
0 & 1
\end{pmatrix}
\]

The common block preconditioners for Stokes require only options:

- `pc_type fieldsplit`
- `pc_field_split_type multiplicity`
- `fieldsplit_0_pc_type hypre`
- `fieldsplit_0_ksp_type preonly`
- `fieldsplit_1_pc_type jacobi`
- `fieldsplit_1_ksp_type preonly`

(\[
P, C = \begin{pmatrix} \hat{A} & B \\ 0 & I \end{pmatrix}
\]

Stokes example

The common block preconditioners for Stokes require only options:

- `pc_type fieldsplit`
- `pc_field_split_type schur`
- `fieldsplit_0_pc_type gamg`
- `fieldsplit_0_ksp_type preonly`
- `fieldsplit_1_pc_type none`
- `fieldsplit_1_ksp_type minres`
- `pc_fieldsplit_schur_factorization_type diag`

Stokes example

The common block preconditioners for Stokes require only options:

\[ PC \left( \begin{array}{cc} \hat{A} & 0 \\ B^T & \hat{S} \end{array} \right) \]

- `pc_type fieldsplit`
- `pc_field_split_type schur`
- `fieldsplit_0_pc_type gamg`
- `fieldsplit_0_ksp_type preonly`
- `fieldsplit_1_pc_type none`
- `fieldsplit_1_ksp_type minres`
- `pc_fieldsplit_schur_factorization_type lower`

The common block preconditioners for Stokes require only options:

- `pc_type fieldsplit`
- `pc_field_split_type schur`
- `fieldsplit_0_pc_type gamg`
- `fieldsplit_0_ksp_type preonly`
- `fieldsplit_1_pc_type none`
- `fieldsplit_1_ksp_type minres`
- `pc_fieldsplit_schur_factorization_type upper`

The common block preconditioners for Stokes require only options:

\[-pc\text{-type} \text{ fieldsplit} \]
\[-pc\text{-field\_split\_type} \text{ schur} \]
\[-\text{fieldsplit\_0\_pc\_type} \text{ gamg} \]
\[-\text{fieldsplit\_0\_ksp\_type} \text{ preonly} \]
\[-\text{fieldsplit\_1\_pc\_type} \text{ lsc} \]
\[-\text{fieldsplit\_1\_ksp\_type} \text{ minres} \]
\[-pc\text{-field\_split\_schur\_factorization\_type} \text{ upper} \]

**PC**
\[
\begin{pmatrix}
\hat{A} & B \\
0 & \hat{S}_{\text{LSC}}
\end{pmatrix}
\]

The common block preconditioners for Stokes require only options:

- `pc_type fieldsplit`
- `pc_field_split_type schur`
- `pc_fieldsplit_schur_factorization_type full`

\[
\begin{pmatrix}
I & 0 \\
B^T A^{-1} & I
\end{pmatrix}
\begin{pmatrix}
\hat{A} & 0 \\
0 & \hat{S}
\end{pmatrix}
\begin{pmatrix}
I & A^{-1} B \\
0 & I
\end{pmatrix}
\]
Why use FGMRES?

Flexible GMRES (FGMRES) allows a different preconditioner at each step:

- Takes twice the memory
- Needed for iterative PCs
- Avoided sometimes with a careful PC choice
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Looks nice,
But can you do this for a real PyLith Example?
First, we try LU on the whole problem `solver_lu.cfg`

```plaintext
[pylithapp.petsc]
snes_view = true
pc_type = lu
```

FAIL

This is due to the saddle point introduced to handle the fault.
First, we try LU on the whole problem `solver_lu.cfg`

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[pylithapp.petsc]
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```

FAIL

This is due to the saddle point introduced to handle the fault.
Next, we split fields using `PC_FIELDSPPLIT` in `solver_fault_additive.cfg`

```py
[pylithapp.timedependent.formulation]
matrix_type = aij
split_fields = True

[pylithapp.petsc]
ksp_max_it = 1000
fs_pc_type = fieldsplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = additive
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_ksp_type = preonly
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
```

Converges in 58 iterations because preconditioner is not that strong.
Next, we split fields using `PC_FIELDSSPLIT`:

`solver_fault_additive.cfg`

```python
[pylithapp.timedependent.formulation]
matrix_type = aij
split_fields = True

[pylithapp.petsc]
ksp_max_it = 1000
fs_pc_type = fieldssplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = additive
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_ksp_type = preonly
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
```

Converges in 58 iterations because preconditioner is not that strong
We need to use a full Schur factorization `solver_fault_exact.cfg`

```Ini
[pylithapp.petsc]
fs_pc_type = fieldsplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = schur
fs_pc_fieldsplit_schur_factorization_type = full
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
fs_fieldsplit_lagrange_multiplier_ksp_type = gmres
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-11
```

Works in one iterate! This is good for checking the physics.
We need to use a full Schur factorization `solver_fault_exact.cfg`

```plaintext
[pylithapp.petsc]
fs_pc_type = fieldsplit
fs_pc_use_amat = true
fs_pc_fieldsplit_type = schur
fs_pc_fieldsplit_schur_factorization_type = full
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = lu
fs_fieldsplit_lagrange_multiplier_pc_type = jacobi
fs_fieldsplit_lagrange_multiplier_ksp_type = gmres
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-11
```

Works in one iterate! This is good for checking the physics.
We can add a user defined preconditioner for the Schur complement

```python
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True
[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```
We can add a user defined preconditioner for the Schur complement

```
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True
[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```

The initial convergence

```
0  SNES  Function norm 1.547533880440e-02
   Linear solve converged due to CONVERGED_RTOL iterations 30
0  KSP  Residual norm 1.158385264202e-02
   Linear solve converged due to CONVERGED_RTOL iterations 30
1  KSP  Residual norm 2.231623131220e-13
   Linear solve converged due to CONVERGED_RTOL iterations 1
1  SNES  Function norm 1.146037096697e-13
```
We can add a user defined preconditioner for the Schur complement

```
[pylithapp.timedependent.formulation]
use_custom_constraint_pc = True

[pylithapp.petsc]
fs_pc_fieldsplit_schur_precondition = user
```

improves to `solver_fault_schur_custompc.cfg`

0  SNES Function norm 1.547533880440e-02
  Linear solve converged due to CONVERGED_RTOL iterations 24
0  KSP Residual norm 1.158385264203e-02
  Linear solve converged due to CONVERGED_RTOL iterations 25
1  KSP Residual norm 5.404403812155e-14
  Linear solve converged due to CONVERGED_RTOL iterations 1
1  SNES Function norm 2.201265688755e-14

and gets much better for larger problems.
You can back off the Schur complement tolerance

```
ksp_type = fgmres
fs_fieldsplit_lagrange_multiplier_ksp_rtol = 1.0e-05
```

at the cost of more iterates

```
0 SNES Function norm 1.547533880440e-02
  0 KSP Residual norm 1.547533880440e-02
Linear solve converged due to CONVERGED_RTOL iterations 10
1 KSP Residual norm 9.761444929927e-08
Linear solve converged due to CONVERGED_RTOL iterations 15
2 KSP Residual norm 4.058177976336e-13
Linear solve converged due to CONVERGED_RTOL iterations 2
1 SNES Function norm 2.763748407804e-13
```
You can back off the primal LU solver

```
fs_fieldsplit_displacement_ksp_type = preonly
fs_fieldsplit_displacement_pc_type = gamg
```

at the cost of more iterates

0  SNES Function norm  1.547533880440e-02  
0  KSP Residual norm  1.547533880440e-02  
Linear solve converged due to CONVERGED_RTOL iterations 12
1  KSP Residual norm  3.659593456893e-04  
Linear solve converged due to CONVERGED_RTOL iterations 15
2  KSP Residual norm  9.111591440754e-06  
Linear solve converged due to CONVERGED_RTOL iterations 16
   
6  KSP Residual norm  3.526238448332e-12  
Linear solve converged due to CONVERGED_RTOL iterations 17
7  KSP Residual norm  8.640836102392e-14  
Linear solve converged due to CONVERGED_RTOL iterations 7
1  SNES Function norm  8.641267905609e-14
You can restore the behavior with a lower tolerance

```
fs_fieldsplit_displacement_ksp_type = gmres
fs_fieldsplit_displacement_ksp_rtol = 5.0e-10
```

but it is quite sensitive to the tolerance.

```
0  SNES Function norm 1.547533880440e-02
  0  KSP Residual norm 1.547533880440e-02
Linear solve converged due to CONVERGED_RTOL iterations 10
  1  KSP Residual norm 9.761445192979e-08
Linear solve converged due to CONVERGED_RTOL iterations 15
  2  KSP Residual norm 7.227466516039e-13
Linear solve converged due to CONVERGED_RTOL iterations 2
  1  SNES Function norm 2.391873654238e-13
```
Nonlinear Systems

Driven Cavity Problem

SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e2
  -da_grid_x 16 -da_grid_y 16 -da_refine 2
  -snes_monitor_short -snes_converged_reason -snes_view
Driven Cavity Problem

SNES ex19.c

./ex19  -lidvelocity 100  -grashof 1e2
    -da_grid_x 16  -da_grid_y 16  -da_refine 2
    -snes_monitor_short -snes_converged_reason -snes_view

lid velocity = 100, prandtl # = 1, grashof # = 100

0  SNES Function norm  768.116
1  SNES Function norm  658.288
2  SNES Function norm  529.404
3  SNES Function norm  377.51
4  SNES Function norm  304.723
5  SNES Function norm  2.59998
6  SNES Function norm  0.00942733
7  SNES Function norm  5.20667e-08

Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e4
    -da_grid_x 16 -da_grid_y 16 -da_refine 2
    -snes_monitor_short -snes_converged_reason -snes_view
Driven Cavity Problem

SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e4
   -da_grid_x 16 -da_grid_y 16 -da_refine 2
   -snes_monitor_short -snes_converged_reason -snes_view

lid velocity = 100, prandtl # = 1, grashof # = 10000
  0 SNES Function norm 785.404
  1 SNES Function norm 663.055
  2 SNES Function norm 519.583
  3 SNES Function norm 360.87
  4 SNES Function norm 245.893
  5 SNES Function norm 1.8117
  6 SNES Function norm 0.00468828
  7 SNES Function norm 4.417e-08
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e5
    -da_grid_x 16 -da_grid_y 16 -da_refine 2
    -snes_monitor_short -snes_converged_reason -snes_view
Nonlinear Systems

Driven Cavity Problem

SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e5
   -da_grid_x 16 -da_grid_y 16 -da_refine 2
   -snes_monitor_short -snes_converged_reason -snes_view

lid velocity = 100, prandtl # = 1, grashof # = 100000
0 SNES Function norm 1809.96
Nonlinear solve did not converge due to DIVERGED_LINEAR_SOLVE iterations 0
Driven Cavity Problem

SNES ex19.c

./ex19 -lidvelocity 100 -grashof 1e5
   -da_grid_x 16 -da_grid_y 16 -da_refine 2 -pc_type lu
   -snes_monitor_short -snes_converged_reason -snes_view

lid velocity = 100, prandtl # = 1, grashof # = 100000

0 SNES Function norm 1809.96
1 SNES Function norm 1678.37
2 SNES Function norm 1643.76
3 SNES Function norm 1559.34
4 SNES Function norm 1557.6
5 SNES Function norm 1510.71
6 SNES Function norm 1500.47
7 SNES Function norm 1498.93
8 SNES Function norm 1498.44
9 SNES Function norm 1498.27
10 SNES Function norm 1498.18
11 SNES Function norm 1498.12
12 SNES Function norm 1498.11
13 SNES Function norm 1498.11
14 SNES Function norm 1498.11
...
Why isn’t SNES converging?

- The Jacobian is wrong (maybe only in parallel)
  - Check with `--snes_check_jacobian` and `--snes_check_jacobian_view`

- The linear system is not solved accurately enough
  - Check with `--pc_type lu`
  - Check `--ksp_monitor_true_residual`, try right preconditioning

- The Jacobian is singular with inconsistent right side
  - Use `MatNullSpace` to inform the KSP of a known null space
  - Use a different Krylov method or preconditioner

- The nonlinearity is just really strong
  - Run with `--info` or `--snes_ls_monitor` to see line search
  - Try using trust region instead of line search `--snes_type tr`
  - Try grid sequencing if possible `--snes_grid_sequence`
  - Use a continuation
**Nonlinear Preconditioning**

**PC preconditions** KSP  
-ksp_type gmres

-pc_type richardson

**SNES preconditions** SNES  
-snes_type ngmres

-npc_snes_type nrichardson
Nonlinear Preconditioning

**PC preconditions**  **KSP**
- `-ksp_type gmres`
- `-pc_type richardson`

**SNES preconditions**  **SNES**
- `-snes_type ngmres`
- `-npc_snes_type nrichardson`
Nonlinear Systems

Nonlinear Use Cases

Warm start Newton

- `snes_type newtonls`
- `npc_snes_type nrichardson -npc_snes_max_it 5`

Cleanup noisy Jacobian

- `snes_type ngmres -snes_ngmres_m 5`
- `npc_snes_type newtonls`

Additive-Schwarz Preconditioned Inexact Newton

- `snes_type aspin -snes_npc_side left`
- `npc_snes_type nasm -npc_snes_nasm_type restrict`
Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type newtonls -snes_converged_reason -pc_type lu

lid velocity = 100, prandtl # = 1, grashof # = 50000
0 SNES Function norm 1228.95
1 SNES Function norm 1132.29
2 SNES Function norm 1026.17
3 SNES Function norm 925.717
4 SNES Function norm 924.778
5 SNES Function norm 836.867
...
21 SNES Function norm 585.143
22 SNES Function norm 585.142
23 SNES Function norm 585.142
24 SNES Function norm 585.142
...
Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type fas -snes_converged_reason -fas_levels_snes_type gs -fas_levels_snes_max_it 6

lid velocity = 100, prandtl # = 1, grashof # = 50000
0 SNES Function norm 1228.95
1 SNES Function norm 574.793
2 SNES Function norm 513.02
3 SNES Function norm 216.721
4 SNES Function norm 85.949
Nonlinear solve did not converge due to DIVERGED_INNER iterations 4
Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
  -snes_type fas -snes_converged_reason
  -fas_levels_snes_type gs -fas_levels_snes_max_it 6
  -fas_coarse_snes_converged_reason

lid velocity = 100, prandtl # = 1, grashof # = 50000

0 SNES Function norm 1228.95
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 12
1 SNES Function norm 574.793
  Nonlinear solve did not converge due to DIVERGED_MAX_IT its 50
2 SNES Function norm 513.02
  Nonlinear solve did not converge due to DIVERGED_MAX_IT its 50
3 SNES Function norm 216.721
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 22
4 SNES Function norm 85.949
  Nonlinear solve did not converge due to DIVERGED_LINE_SEARCH its 42
Nonlinear solve did not converge due to DIVERGED_INNER iterations 4
Nonlinear Systems

Nonlinear Preconditioning

```bash
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type fas -snes_converged_reason -fas_levels_snes_type gs -fas_levels_snes_max_it 6 -fas_coarse_snes_linesearch_type basic -fas_coarse_snes_converged_reason
```

```
lid velocity = 100, prandtl # = 1, grashof # = 50000
  0 SNES Function norm 1228.95
    Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
  ...
  47 SNES Function norm 78.8401
    Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 5
  48 SNES Function norm 73.1185
    Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
  49 SNES Function norm 78.834
    Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 5
  50 SNES Function norm 73.1176
    Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
  ...
```
Nonlinear Systems

Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
-snes_type nrichardson -npc_snes_max_it 1 -snes_converged_reason
-npc_snes_type fas -npc_fas_coarse_snes_converged_reason
-npc_fas_levels_snes_type gs -npc_fas_levels_snes_max_it 6
-npc_fas_coarse_snes_linesearch_type basic

lid velocity = 100, prandtl $\# = 1$, grashof $\# = 50000$

0 SNES Function norm 1228.95
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
1 SNES Function norm 552.271
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 27
2 SNES Function norm 173.45
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 45

43 SNES Function norm 3.45407e-05
  Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
44 SNES Function norm 1.6141e-05
  Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
45 SNES Function norm 9.13386e-06
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 45
Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type ngmres -npc_snes_max_it 1 -snes_converged_reason -npc_snes_type fas -npc_fas_coarse_snes_converged_reason -npc_fas_levels_snes_type gs -npc_fas_levels_snes_max_it 6 -npc_fas_coarse_snes_linesearch_type basic

lid velocity = 100, prandtl # = 1, grashof # = 50000

0 SNES Function norm 1228.95
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 6
1 SNES Function norm 538.605
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 13
2 SNES Function norm 178.005
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 24

... 27 SNES Function norm 0.000102487
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE its 2
28 SNES Function norm 4.2744e-05
  Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 2
29 SNES Function norm 1.01621e-05
  Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 29
```
./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short
-snes_type ngmres -npc_snes_max_it 1 -snes_converged_reason
-npc_snes_type fas -npc_fas_coarse_snes_converged_reason
-npc_fas_levels_snes_type newtonls -npc_fas_levels_snes_max_it 6
-npc_fas_levels_snes_linesearch_type basic
-npc_fas_levels_snes_max_linear_solve_fail 30
-npc_fas_levels_ksp_max_it 20 -npc_fas_levels_snes_converged_reason
-npc_fas_coarse_snes_linesearch_type basic

lid velocity = 100, prandtl # = 1, grashof # = 50000

0 SNES Function norm 1228.95
   Nonlinear solve did not converge due to DIVERGED_MAX_IT its 6

...

Nonlinear solve converged due to CONVERGED_SNORM_RELATIVE its 1

...

1 SNES Function norm 0.1935
2 SNES Function norm 0.0179938
3 SNES Function norm 0.00223698
4 SNES Function norm 0.000190461
5 SNES Function norm 1.6946e-06
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 5
```
Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type composite -snes_composite_type additiveoptimal -snes_composite_sneses fas,newtonls -snes_converged_reason -sub_0_fas_levels_snes_type gs -sub_0_fas_levels_snes_max_it 6 -sub_0_fas_coarse_snes_linesearch_type basic -sub_1_snes_linesearch_type basic -sub_1_pc_type mg

lid velocity = 100, prandtl # = 1, grashof # = 50000

0 SNES Function norm 1228.95
1 SNES Function norm 541.462
2 SNES Function norm 162.92
3 SNES Function norm 48.8138
4 SNES Function norm 11.1822
5 SNES Function norm 0.181469
6 SNES Function norm 0.00170909
7 SNES Function norm 3.24991e-08

Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 7
Nonlinear Systems

Nonlinear Preconditioning

./ex19 -lidvelocity 100 -grashof 5e4 -da_refine 4 -snes_monitor_short -snes_type composite -snes_composite_type multiplicative -snes_composite_sneses fas,newtonls -snes_converged_reason -sub_0_fas_levels_snes_type gs -sub_0_fas_levels_snes_max_it 6 -sub_0_fas_coarse_snes_linesearch_type basic -sub_1_snes_linesearch_type basic -sub_1_pc_type mg

lid velocity = 100, prandtl # = 1, grashof # = 50000

0 SNES Function norm 1228.95
1 SNES Function norm 544.404
2 SNES Function norm 18.2513
3 SNES Function norm 0.488689
4 SNES Function norm 0.000108712
5 SNES Function norm 5.68497e-08
Nonlinear solve converged due to CONVERGED_FNORM_RELATIVE iterations 5
## Nonlinear Preconditioning

<table>
<thead>
<tr>
<th>Solver</th>
<th>T</th>
<th>N. It</th>
<th>L. It</th>
<th>Func</th>
<th>Jac</th>
<th>PC</th>
<th>NPC</th>
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<tbody>
<tr>
<td>$(\nabla \setminus K - MG)$</td>
<td>9.83</td>
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<td>352</td>
<td>34</td>
<td>85</td>
<td>370</td>
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<td>231</td>
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<td>$(\nabla \setminus K - MG)$</td>
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<td>0</td>
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<td>377</td>
<td>754</td>
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<td>FAS</td>
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<td>232</td>
<td>90</td>
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<tr>
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<td>103</td>
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<td>NGMRES $- R$ FAS</td>
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</tbody>
</table>
See discussion in:


For any other solver problems, contact cig-short@geodynamics.org